Closing tonight (11pm): 10.1
Closing Fri: 2.1, 2.2, 2.3

Entry Task: Draw rough sketches of

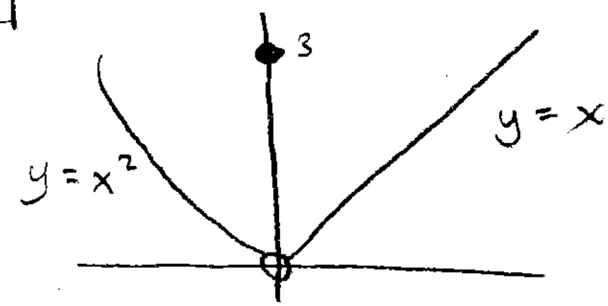
1. $h(x) = \begin{cases} x^2 & \text{, if } x < 0; \\ 3 & \text{, if } x = 0; \\ x & \text{, if } x > 0. \end{cases}$

2. $g(x) = \frac{1}{x^2}$

3. $j(x) = \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{(x-3)}$

4. $f(x) = \frac{|x|}{x} = \begin{cases} \frac{-x}{x}, & \text{if } x<0; \\ \frac{x}{x}, & \text{if } x \geq 0. \end{cases}$

$\boxed{1}$



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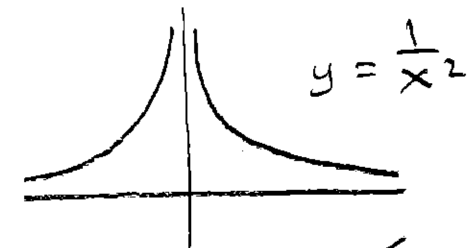

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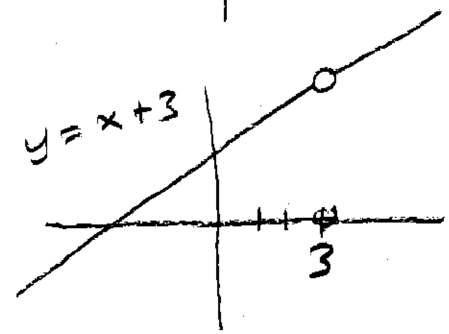

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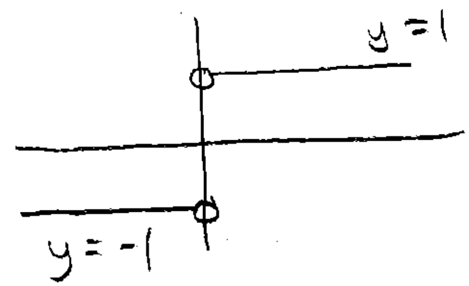


2.2 Limits

$$\lim_{x \to a} f(x) = L$$

"the **limit** of f(x), as x approaches a, is L". It means

as *x* takes on values closer and closer to *a*,

y = f(x) takes on values closer and closer to L.

Find

$h(0) =$ 3

$\lim_{x \to 0} h(x) =$ 0

$f(0) =$ DNE

$\lim_{x \to 0} f(x) =$ DNE

$g(0) =$ DNE

$\lim_{x \to 0} g(x) = \infty$

$\lim_{x \to \infty} g(x) =$ 0

$\lim_{x \to -\infty} g(x) =$ 0

$j(3) =$ DNE

$\lim_{x \to 3} j(x) =$ 6

One-sided limits

$$\lim_{x \to a^-} f(x) = L$$

"the limit of *f(x)*, as *x* approaches *a* **from the left**, is *L*". It means
as *x* takes on values closer to and **from the left** (smaller values) of *a*, *y = f(x)* takes on values closer and closer to L.

$$\lim_{x \to a^+} f(x) = L$$

"the limit of *f(x)*, as *x* approaches *a* **from the right**, is *L*".

Note:

$$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \text{both} \begin{cases} \lim_{x \to a^-} f(x) = L \\ \lim_{x \to a^+} f(x) = L \end{cases}$$

Find

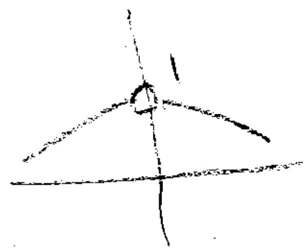
$\lim_{x \to 0^-} f(x) =$ -1

$\lim_{x \to 0^+} f(x) =$ 1

What if we can't easily graph?
We can try plugging in points (but be careful).

Example: Find

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$



x	0.1	-0.1	0.01	-0.01
$\frac{\sin(x)}{x}$	0.9983	0.9983	0.9999998	0.9999998

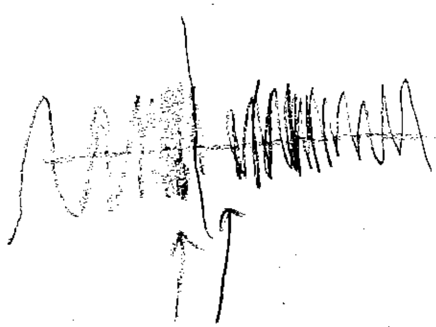
LOOKS LIKE IT IS APPROACHING 1

Example: Find

$$\lim_{x\to 0} \sin\left(\frac{\pi}{x}\right) = \text{DNE}$$

x	0.1	0.01	0.003	0.000009
$\sin\left(\frac{\pi}{x}\right)$	0	0	-0.866	-0.34202

NOT APPROACHING



OSCILLATES INFINITELY OFTEN AROUND $x=0$.

2.3 Limit Laws and Strategies

Some Basic Limit Laws:

1. $\lim_{x \to a} c = c$

2. $\lim_{x \to a} x = a$

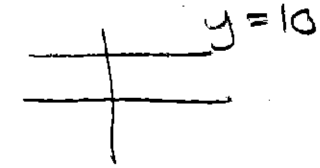
3. $\lim_{x \to a} [f(x) + g(x)]$
$$= \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

4. $\lim_{x \to a} [f(x) g(x)]$
$$= \lim_{x \to a} f(x) \lim_{x \to a} g(x)$$

5. If $\boldsymbol{\lim_{x \to a} g(x) \neq 0}$, then
$$\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

Examples:

1. $\lim_{x \to -7} 10 = 10$



2. $\lim_{x \to 14} x = 14$

3. $\lim_{x \to -2} [x + 6] = \lim_{x \to -2} x + \lim_{x \to -2} 6$
$$= -2 + 6 = 4$$

4. $\lim_{x \to 5} [2x^2] = \lim_{x \to 5} 2 \lim_{x \to 5} x \lim_{x \to 5} x$
$$= 2 \cdot 5 \cdot 5 = 50$$

5. $\lim_{x \to 4} \left[\frac{x + 2}{x^2}\right] = \frac{\lim_{x \to 4} (x + 2)}{\lim_{x \to 4} x^2}$
$$= \frac{4+2}{4^2} = \frac{6}{16} = \frac{3}{8}$$

Limit Flow Chart for

$$\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right]$$

1. Try plugging in the value.
   If denominator ≠ 0, done!

2. **If denom = 0 & numerator ≠ 0**, the answer is -∞, +∞ or DNE. Examine the sign (pos/neg) of the output from each side.

3. **If denom = 0 & numerator = 0**, Use algebraic methods to simplify and cancel until one of them is not zero.

Examples:

1. $\lim_{x \to 1} \frac{x+6}{x-4} = \boxed{\frac{7}{-3}}$ DONE!

$\frac{6}{0}$ NUM + DEN +

2a. $\lim_{x \to 2^+} \frac{x+4}{x-2} = \boxed{+\infty}$

NUM + DEN −

2b. $\lim_{x \to 2^-} \frac{x+4}{x-2} = \boxed{-\infty}$

2c. $\lim_{x \to 2} \frac{x+4}{x-2} = \boxed{\text{DNE}}$ ← DIFFERENT

$\frac{1+1}{0}$ NUM + DEN +

2d. $\lim_{x \to 0} \frac{\cos(x)+e^x}{x^2} = \boxed{+\infty}$

For the den = 0, num = 0 case, here is a summary of some algebra to try:

Strategy 1: Factor/Cancel
Strategy 2: Simplify Fractions
Strategy 3: Expand/Simplify
Strategy 4: Multiply by Conjugate

Strategy 5: Change Variable
Strategy 6: Compare to other functions (Squeeze Thm)

Examples:

1. $\lim_{x\to 5} \frac{x^2 - 25}{x - 5} = \lim_{x \to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}}$

$= 5 + 5 = \boxed{10}$

2. $\lim_{h\to 0} \frac{\left(\frac{1}{2+h} - \frac{1}{2}\right)}{h} \cdot \frac{(2)(2+h)}{(2)(2+h)}$

$= \lim_{h \to 0} \frac{2 - (2+h)}{h(2)(2+h)}$

$= \lim_{h \to 0} \frac{\cancel{2} - \cancel{2} - h}{h(2)(2+h)}$

$= \lim_{h \to 0} \frac{-1}{2(2+h)} = \frac{-1}{2(2+0)} = \boxed{-1/4}$

3. $\lim_{h\to 0} \frac{(3+h)^2-9}{h} =$

$\lim_{h\to 0} \frac{\cancel{9}+6h+h^2-\cancel{9}}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(6+h)}{\cancel{h}}$

$= 6+0 = \boxed{6}$

4. $\lim_{x\to 4} \frac{(x-4)}{(\sqrt{x}-2)} = \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)}$

$= \lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{(x-4)}}$

$= \sqrt{4}+2 = 2+2 = \boxed{4}$

Squeeze Thm:

If the following hold:

(1) $g(x) \le f(x) \le h(x)$ near $x = a$

(2) $\lim_{x\to a} g(x) = L$ and $\lim_{x\to a} h(x) = L$

then

$$\lim_{x\to a} f(x) = L$$

Example: Find

$$\lim_{x\to 0} x^2 \cos\left(\frac{10}{x}\right) =$$

NOTE

$$-1 \le \cos\left(\frac{10}{x}\right) \le 1$$

$$\Rightarrow -x^2 \le x^2\cos\left(\frac{10}{x}\right) \le x^2$$

AND since

$$\lim_{x\to 0} -x^2 = 0 \quad \text{AND}$$

$$\lim_{x\to 0} x^2 = 0$$

WE HAVE

$$\boxed{\lim_{x\to 0} x^2\cos\left(\frac{10}{x}\right) = 0}$$